2E1.  Which of the expressions below correspond to the statement: the probability of rain on Monday?

**(2) or (4) can be correct**

2E2.  Which of the following statements corresponds to the expression: Pr(Monday|rain)?

**(3) The probability that it is Monday, given that it is raining.**

2E3. Which of the expressions below correspond to the statement: the probability that it is Monday, given that it is raining?

**(1) Pr(Monday|rain)**

2E4. The Bayesian statistician Bruno de Finetti (1906–1985) began his book on probability theory with the declaration: “PROBABILITY DOES NOT EXIST.” The capitals appeared in the original, so I imagine de Finetti wanted us to shout this statement. What he meant is that probability is a device for describing uncertainty from the perspective of an observer with limited knowledge; it has no objective reality. Discuss the globe tossing example from the chapter, in light of this statement. What does it mean to say “the probability of water is 0.7”?

**The reason why probability is just describing uncertainty is because if we had all of the information at hand, we would predict the globe toss 100% of the time. We could take many variables into play such as the flick speed, wind, finger width, etc., but we don’t know all of the variables in this very earth. “The probability of water is 0.7” is measuring the degree of uncertainty versus the outcome given we don’t know all of the information.**

2M1. Recall the globe tossing model from the chapter. Compute and plot the grid approximate posterior distribution for each of the following sets of observations. In each case, assume a uniform prior for p.

(1) W,W,W



(2) W,W,W,L



(3) L,W,W,L,W,W,W



2M2. Now assume a prior for p that is equal to zero when p < 0.5 and is a positive constant when p ≥ 0.5. Again compute and plot the grid approximate posterior distribution for each of the sets of observations in the problem just above.

1.



2.



3.



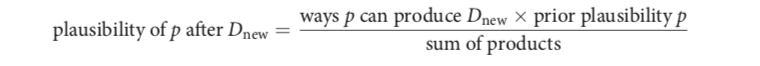
2M3. Suppose there are two globes, one for Earth and one for Mars. The Earth globe is 70% covered in water. The Mars globe is 100% land. Further suppose that one of these globes—you don’t know which—was tossed in the air and produced a “land” observation. Assume that each globe was equally likely to be tossed. Show that the posterior probability that the globe was the Earth, conditional on seeing “land” (Pr(Earth|land)), is 0.23.

P(Land|Earth) = .3

P(Land|Mars) = 1.0

P(Earth) = .5

P(Mars) = .5



P(Earth|land) = 0.3 \* 0.5 /(0.3 \* 0.5 + 1 \* 0.5)

**0.2307692**

2M4. Suppose you have a deck with only three cards. Each card has two sides, and each side is either black or white. One card has two black sides. The second card has one black and one white side. The third card has two white sides. Now suppose all three cards are placed in a bag and shuffled. Someone reaches into the bag and pulls out a card and places it flat on a table. A black side is shown facing up, but you don’t know the color of the side facing down. Show that the probability that the other side is also black is 2/3. Use the counting method (Section 2 of the chapter) to approach this problem. This means counting up the ways that each card could produce the observed data (a black side facing up on the table).

Cards = 3

Card 1 = 2 black

Card 2 = 1 black, 1 white

Card 3 = 2 white

Sides = front, back (2)

Color = black, white (2)

**All combinations**

(Black[front], Black[back])

(Black[back], Black[front])

(Black[front], White[back])

(White[front], Black[back])

(White[front], White[back])

(White[back], White[front])

**Given that 1 side is black, there are 2 black options and 1 white. Hence, there is a 2/3 chance the other side is black.**

2M5. Now suppose there are four cards: B/B, B/W, W/W, and another B/B. Again suppose a card is drawn from the bag and a black side appears face up. Again calculate the probability that the other side is black.

Cards = 4

Card 1 = 2 black

Card 2 = 1 black, 1 white

Card 3 = 2 white

Card 4 = 2 black

**Card1**

(Black[front], Black[back])

(Black[back], Black[front])

**Card2**

(Black[front], White[back])

(White[front], Black[back])

**Card3**

(White[front], White[back])

(White[back], White[front])

**Card4**

(Black[front], Black[back])

(Black[back], Black[front])

**There is a 4/5 chance that the other side of the card is black.**

2M6. Imagine that black ink is heavy, and so cards with black sides are heavier than cards with white sides. As a result, it’s less likely that a card with black sides is pulled from the bag. So again assume there are three cards: B/B, B/W, and W/W. After experimenting a number of times, you conclude that for every way to pull the B/B card from the bag, there are 2 ways to pull the B/W card and 3 ways to pull the W/W card. Again suppose that a card is pulled and a black side appears face up. Show that the probability the other side is black is now 0.5. Use the counting method, as before.

Card1

(Black[front], Black[back])

(Black[back], Black[front]) = 2

Card2

(Black[front], White[back]) \* 2 = 2

(White[front], Black[back])

**Since there are 2 ways to get the B/W card, there are now 4 possible combinations. 2 of those are B/W and the other 2 are B/B. Therefore, there is a 2/4 chance and the probability is .5.**

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2M7. Assume again the original card problem, with a single card showing a black side face up. Before looking at the other side, we draw another card from the bag and lay it face up on the table. The face that is shown on the new card is white. Show that the probability that the first card, the one showing a black side, has black on its other side is now 0.75. Use the counting method, if you can. Hint: Treat this like the sequence of globe tosses, counting all the ways to see each observation, for each possible first card.

Cards = 3

Card 1 = 2 black

Card 2 = 1 black, 1 white

Card 3 = 2 white

Sides = front, back (2)

Color = black, white (2)

**All options**

**Color of first Card Color of Second Card**

**B/B Options White options**

(Black[front], Black[back]) (Black[front], White[back])

(Black[front], Black[back]) (White[front], Black[back])

(Black[front], Black[back]) (White[front], White[back])

(Black[front], Black[back]) (White[back], White[front])

(Black[back], Black[front]) (Black[front], White[back])

(Black[back], Black[front]) (White[front], Black[back])

(Black[back], Black[front]) (White[front], White[back])

(Black[back], Black[front]) (White[back], White[front])

**B/W Options**

(Black[front], White[back]) (Black[front], Black[back])

(Black[front], White[back]) (Black[back], Black[front])

(Black[front], White[back]) (White[front], White[back])

(Black[front], White[back]) (White[back], White[front])

**There are 8 total options where the first card has a side that is black and the second card has a side that is white. Of the 8, there are only 6 possibilities where the other side of the black card is also black. Therefore, 6/8 = .75**